

5. You deposit \$300 in an account earning 5% interest compounded annually. How much will you have in the account in 10 years?

$$P=300, R=5\%, t=10$$

$$A=P(1+R)^n$$

$$A=300(1+0.05)^{10}=488.7$$

6. How much will \$1000 deposited in an account earning 7% interest compounded annually be worth in 20 years?

$$P=1000, R=7\%, n=20$$

$$A=P(1+R)^n$$

$$A=1000(1.07)^{20}=8769.68$$

7. You deposit \$2000 in an account earning 3% interest compounded monthly

- a. How much will you have in the account in 20 years?

$$P=2000$$

$$R=3/12=0.25\%$$

$$n=20*12=240$$

$$A=P(1+R)^n \gg A=2000(1.0025)^{240}=3641.51$$

- b. How much interest will you earn?  $3641.51-2000=1641.51$

8. You deposit \$10,000 in an account earning 4% interest compounded monthly. a. How much will you have in the account in 25 years? b. How much interest will you earn?

$$P=10000, R=4/12=0.333\%, n=25*12=300$$

$$A=P(1+R)^n=10000(1.00333)^{300}=26868.52$$

$$\text{Interest}=A-P=26868.52-10000=16868.52$$

9. How much would you need to deposit in an account now in order to have \$6,000 in the account in 8 years? Assume the account earns 6% interest compounded monthly.

$$A=6000$$

$$N=8*12=96$$

$$R=6/12=0.5\% \quad P=6000/(1.0050^96)=3717.14$$

10. You deposit \$200 each month into an account earning 3% interest compounded monthly.

a. How much will you have in the account in 30 years?  $P_N=200((1+0.03/12)^{30*12}-1)/0.03/12=116547.4$

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$$0.03/12$$

b. How much total money will you put into the account? c. How much total interest will you earn?  $TOTAL\ DEPOSIT=200*360=72000$

$$TOTAL\ INTEREST=116547.4-72000=44547.4$$

13. Jose has determined he needs to have \$800,000 for retirement in 30 years. His account earns 6% interest.

a. How much would he need to deposit in the account each month?

$$800000 = \frac{d((1 + \frac{0.06}{12})^{30(12)} - 1)}{\frac{0.06}{12}}$$

$$800000 = \frac{d((1.005)^{360} - 1)}{0.005}$$

$$800000 = \frac{d(6.022575212 - 1)}{0.005}$$

$$800000 = \frac{d(5.022575212)}{0.005}$$

$$800000 = \frac{d(5.022575212)}{0.005}$$

$$800000 = d(1004.515042)$$

$$d = \frac{800000}{1004.515042}$$

$$d = 796.4042016$$

b. How much total money will he put into the account?  $796.40 \times 360 = 286704$

c. How much total interest will he earn?  $800000 - 286704 = 513296$

15. You want to be able to withdraw \$30,000 each year for 25 years. Your account earns 8% interest.

a. How much do you need in your account at the beginning?

$$P_0 = \frac{30000(1 - (1 + 0.08)^{-25})}{0.08} = 320243.29$$

b. How much total money will you pull out of the account?  $30000 \times 25 = 750000$

c. How much of that money is interest?  $750000 - 320243.29 = 429756.71$

17. You have \$500,000 saved for retirement. Your account earns 6% interest. How much will you be able to pull out each month if you want to be able to take withdrawals for 20 years?

$$\frac{P(r/12)}{1-(1+r/12)^{-12t}} = \frac{pm=500000(0.005)}{1-(1+0.005)^{-12*20}} = 3582.16$$

19. You can afford a \$700 per month mortgage payment. You've found a 30 year loan at 5% interest.

a. How big of a loan can you afford?  $700 \times 12 \times 30 = 252000$

b. How much total money will you pay the loan company?  
 $252000 + 252000 \times 0.05 = 264600$

c. How much of that money is interest?  $264600 - 252000 = 12600$

21. You want to buy a \$25,000 car. The company is offering a 2% interest rate for 48 months (4 years). What will your monthly payments be?

$$Pm = \frac{25000(0.02/12)}{((1+0.001667)^{48}-1)} = 500.81$$

23. You want to buy a \$200,000 home. You plan to pay 10% as a down payment and take out a 30 year loan for the rest.

How much is the loan amount going to be  
 Loan amount = Purchase price of Home x (1 – Percentage of down payment)

$$= \$200,000 \times (1 - 0.10)$$

$$= \$200,000 \times 0.90$$

$$= \$180,000$$

a.

b. What will your monthly payments be if the interest rate is 5%?

Monthly Interest Rate (n) = 0.416667% per month [5.00% / 12 Months]

Number of months (n) = 360 Months [30 Years x 12 Months]

c.

Therefore, the Monthly Loan Payment =  $[P \times \{r (1 + r)^n\}] / [(1 + r)^n - 1]$

$$[\$180,000 \times \{0.00416667 \times (1 + 0.00416667)^{360}\}] / [(1 + 0.00416667)^{360} - 1]$$

$$= [\$180,000 \times \{0.00416667 \times 4.467744314\}] / [4.467744314 - 1]$$

$$= [\$180,000 \times 0.018615601] / 3.467744314$$

$$= \$3,350.81 / 3.467744314$$

$$= \$966.28 \text{ per month}$$

d. What will your monthly payments be if the interest rate is 6%?

Loan Amount (P) = \$180,000

Monthly Interest Rate (n) = 0.50% per month [6.00% / 12 Months]

Number of months (n) = 360 Months [30 Years x 12 Months]

Therefore, the Monthly Loan Payment =  $[P \times \{r (1 + r)^n\}] / [(1 + r)^n - 1]$

$$= [\$180,000 \times \{0.0050 \times (1 + 0.0050)^{360}\}] / [(1 + 0.0050)^{360} - 1]$$

$$= [\$180,000 \times \{0.0050 \times 6.022575212\}] / [6.022575212 - 1]$$

$$= [\$180,000 \times 0.030112876] / 5.022575212$$

$$= \$5,420.31 / 5.022575212$$

$$= \$1,079.19 \text{ per month}$$

35. Suppose you just discovered that your uncle, who died 100 years ago, had left you \$2000 as a gift so you can attend college.

- a. If your uncle invested that money at 4% annually for 100 years, how much would you have today?

$$A = P + prt$$

$$A = 2000(0.04) \cdot 100 + 2000 = 10000$$

- b. Repeat Part (a), using 4% monthly instead.

$$A = 2000 \cdot 0.04 \cdot 1200 + 2000 = 98000$$

- c. Repeat Part (a), using 4% continuously instead. Compare your answers.

$$A = 2000(1.04)^{100} = 101009.89$$

37. You want to invest some money so that in 36 years, you will have \$100,000. Supposing that you have an account earning interest at 5.2% continuously, how much do you have to invest today?

$$A = Pe^{rt} \quad P = A/(e^{rt}) = 100000/(e^{0.052 \cdot 36}) = 15381.57$$

39. Briefly describe the main difference between a “deficit” and a “debt.”

Debt is money that is owed or is due while deficit is the amount by which a sum of money falls short of

40. Give a simple definition for “federal deficit.” Amount by which the government spends bigger money than the money it gets from taxes in a particular year

#### 4.1

1. A ball is drawn randomly from a jar that contains 6 red balls, 2 white balls, and 5 yellow balls. Find the probability of the given event.

- a. A red ball is drawn

$$P(\text{red}) = (\text{number of red balls})/(\text{total number of balls})$$

$$P = 6/(6 + 2 + 5) = 6/13$$

b. A white ball is drawn

$$P(\text{white}) = (\text{number of white balls})/(\text{total number of balls})$$

Assuming no ball has been drawn yet,  
 $P = 2/13$

3. A group of people were asked if they had run a red light in the last year. 150 responded "yes", and 185 responded "no". Find the probability that if a person is chosen at random, they have run a red light in the last year.

$$P = \frac{\text{Total people who had run a green light}}{\text{Total number of people}} = \frac{150}{335} = 0.4478$$

5. Compute the probability of tossing a six-sided die (with sides numbered 1 through 6) and getting a 5.

Total sample space = 6, chance of getting 5 is one

Chance of getting a 5 =  $1/6$

7. Giving a test to a group of students, the grades and gender are summarized below. If one student was chosen at random, find the probability that the student was female.

|        | A  | B  | C  | Total |
|--------|----|----|----|-------|
| Male   | 8  | 18 | 13 | 39    |
| Female | 10 | 4  | 12 | 26    |
| Total  | 18 | 22 | 25 | 65    |

$$P(\text{female}) = 26/65 = 2/5$$

8. Compute the probability of tossing a six-sided die and getting an even number  
Chances of getting an even number is 3  
 $P(\text{even number}) = 3/6 = 1/2$

11. If you pick one card at random from a standard deck of cards, what is the probability it will be a King? Number of king cards = 13  
Total no. of cards = 52  
 $P(\text{king}) = 13/52 = 1/4$

12. Compute the probability of rolling a 12-sided die and getting a number other than 8  
Sample space of 12 sided die = 12

Chance of getting a 8=1

$$P(8)=1/12$$

$$P(8')=1-1/12=11/12$$

15. Referring to the grade table from question #7, what is the probability that a student chosen at random did NOT earn a C?

$$P(c)=25/65=5/13$$

$$(P')=1-5/13=8/13$$

17. A six-sided die is rolled twice. What is the probability of showing a 6 on both rolls?

Probability of not getting a 6 in the first roll =5/6

Probability of not getting a 6 in the second roll=5/6

In both=5/6\*5/6=25/36, therefore p of getting a six=1-25/36=11/36

19. A die is rolled twice. What is the probability of showing a 5 on the first roll and an even number on the second roll?

Probability of not getting a 5 =5/6

Probability of getting an odd number =1/2

In both=5/6\*1/2=5/12. Therefore P=1-5/12=7/12

41. Suppose a math class contains 25 students, 14 females (three of whom speak French) and 11 males (two of whom speak French). Compute the probability that a randomly selected student speaks French, given that the student is female.

Probability of being female=14/25

Probability of speaking French =3/5

Probability therefore=14/25\*3/5=42/125

43. A certain virus infects one in every 400 people. A test used to detect the virus in a person is positive 90% of the time if the person has the virus and 10% of the time if the person does not have the virus. Let A be the event "the person is infected" and B be the event "the person tests positive".

a. Find the probability that a person has the virus given that they have tested positive, i.e. find  $P(A | B)$ .

By the law of complements  $P(A') = 1 - 1/400 = 399/400$

By law of total probability  $P(B) = P(B|A) \cdot P(A) + P(B|A') \cdot P(A')$   
 $9/10 \cdot 1/400 + 1/10 \cdot 399/400 = 51/500$

Using cause's theorem

$$P(A|B) = P(B|A) \cdot P(A) / P(B) = 9/10 \cdot 1/400 \cdot 500 / 51 = 3/136$$

$$P(A) = 1/400$$

$$P(B|A) = 9/10$$

$$P(B|A') = 1/10$$

b. Find the probability that a person does not have the virus given that they test negative, i.e. find  $P(\text{not } A | \text{not } B)$ .

$$P(A'|B') = 1 - 3/316 = 313/316$$

45. A certain disease has an incidence rate of 0.3%. If the false negative rate is 6% and the false positive rate is 4%, compute the probability that a person who tests positive actually has the disease.  $P(D) = 0.003$  and  $p(D') = 0.997$

$$P(P|D) = 0.96, P(P|D') = 0.04$$

$$P(P'|D) = 0.06, P(P'|D') = 0.94$$

$$\text{required probability} = 0.96 \cdot 0.003 / (0.96 \cdot 0.003 + 0.04 \cdot 0.997)$$

$$\text{required probability} = 0.067$$

47. A certain group of symptom-free women between the ages of 40 and 50 are randomly selected to participate in mammography screening. The incidence rate of breast cancer among such women is 0.8%. The false negative rate for the mammogram is 10%. The false positive rate is 7%. If a the mammogram results for a particular

woman are positive (indicating that she has breast cancer), what is the probability that she actually has breast cancer?

False negative rate is 10%

Therefore actually positive from total tested negative =10%

Actually negative from total tested negative=1-10%=0.9

49. A boy owns 2 pairs of pants, 3 shirts, 8 ties, and 2 jackets. How many different outfits can he wear to school if he must wear one of each item?

$$2! \cdot 3! \cdot 8! \cdot 2! = 5806080$$

51. How many three-letter "words" can be made from 4 letters "FGHI" if

a. repetition of letters is allowed  $4 \cdot 4 \cdot 4 = 64$

b. repetition of letters is not allowed  $4! = 4 \cdot 3 \cdot 2 = 24$

53. All of the license plates in a particular state feature three letters followed by three digits (e.g. ABC 123). How many different license plate numbers are available to the state's Department of Motor Vehicles?  $24! / (3! \cdot 9! / 3!) = 6.254 \cdot 10^{27}$

55. A pianist plans to play 4 pieces at a recital. In how many ways can she arrange these pieces in the program?  $4! = 24$

57. Seven Olympic sprinters are eligible to compete in the 4 x 100 m relay race for the USA Olympic team. How many four-person relay teams can be selected from among the seven athletes?  $7! / (7-4)! = 840$

59. In western music, an octave is divided into 12 pitches. For the film Close Encounters of the Third Kind, director Steven Spielberg asked composer John Williams to write a five-note theme, which aliens would use to communicate with people on Earth. Disregarding rhythm and octave changes, how many five-note themes are possible if no note is repeated?

Total number of pitches=12

No.of notes=5

No.of ways of arranging the 12 pitches in a theme of 5 notes without repetition

Number of choices for  $N_1=12$

Number of choices for  $N_2=12-1=11$

,, ,,  $N_3=12-2=10$

,,,, ,,,,  $N_4=12-3=9$

,, ,,  $N_5=12-4=8$

Number of ways therefore  $=12*11*10*9*8=95040$

61. In how many ways can 4 pizza toppings be chosen from 12 available toppings?

$$12!/(12-4)! = 11880$$

63. In the 6/50 lottery game, a player picks six numbers from 1 to 50. How many different choices does the player have if order doesn't matter?

$$N=N!/(N-R)! = 50!/44! = 1.144*10^{10}$$

75. In a lottery game, a player picks six numbers from 1 to 23. If the player matches all six numbers, they win 30,000 dollars. Otherwise, they lose \$1. Find the expected value of this game.  $1/30000=3.33*10^{-5}$

77. A company estimates that 0.7% of their products will fail after the original warranty period but within 2 years of the purchase, with a replacement cost of \$350. If they offer a 2 year extended warranty for \$48, what is the company's expected value of each warranty sold? Company's net  $=48-350=-302$  Expected value  $=(-302*0.007+48*0.993)=45.55$

83. Three people put their names in a hat, then each draw a name, as part of a randomized gift exchange. What is the probability that no one draws their own name? What about with four people? Probability of first person drawing own name  $=1/3$

With 4 people  $=1/4$

91. A die is tossed in a game. Suppose you win \$10 if you get a six and lose \$2 otherwise. Calculate the expected value of this game.

$$\text{Pr } 10 + \text{pr}(0)*0 = 1/6*10 + 0 = 1.667$$

## 5.2

1. A political scientist surveys 28 of the current 106 representatives in a state's congress. Of them, 14 said they were supporting a new education bill, 12 said there were not supporting the bill, and 2 were undecided. a. What is the population of this survey? 106b. What is the size of the population? 28 c. What is the size of the sample? 14/28 d. Give the sample statistic for the proportion of voters surveyed who said they were supporting 50%

Support the education bill. e. Based on this sample, we might expect how many of the representatives to support the education bill?  $0.5(106)=53$

3. Identify the most relevant source of bias in this situation: A survey asks the following: Should the mall prohibit loud and annoying rock music in clothing stores catering to teenagers? **Stacked or driving inquiry is the most significant wellspring of predisposition in the given circumstance, the inquiries wording drives the responder towards an answer, since they are asking "noisy and irritating" which may lead responder towards saying "Shopping center ought to disallow boor and irritating rock music in attire stores providing food".**

5. Identify the most relevant source of bias in this situation: A survey asks people to report their actual income and the income they reported on their IRS tax form. **The survey insinuates that the people have different actual incomes from what they reported in the IRS system which is a red flag for dishonesty**

7. Identify the most relevant source of bias in this situation: A survey asks the following: Should the death penalty be permitted if innocent people might die? **The statement lacks absoluteness because its condition therefore judgement could be one sided**

9. In a study, you ask the subjects their age in years. Is this data qualitative or quantitative? **Quantitative**

11. Does this describe an observational study or an experiment: The temperature on randomly selected days throughout the year was measured. **Observational study**

13. In a study, the sample is chosen by separating all cars by size, and selecting 10 of each size grouping. What is the sampling method? **Random sampling**

15. A team of researchers is testing the effectiveness of a new HPV vaccine. They randomly divide the subjects into two groups. Group 1 receives new HPV vaccine, and Group 2 receives the existing HPV vaccine. The patients in the study do not know which group they are in. a. Which is the treatment group? **Group 1 because they are receiving a new drug** b. Which is the control group (if there is one)? **group 2 because they are not receiving a new drug** c. Is this study blind, double-blind, or neither? **Study is blind because patients do not know which treatment they received.** Is this best described as an experiment, a controlled experiment, or a placebo controlled experiment? **This is a controlled experiment because we are comparing it with a given vaccine does not with placebo**

17. A teacher wishes to know whether the males in his/her class have more conservative attitudes than the females. A questionnaire is distributed assessing attitudes. a. Is this a sampling or a census? census b. Is this an observational study or an experiment? **Observational study** c. Are there any possible sources of bias in this study? **no**

19. A farmer believes that playing Barry Manilow songs to his peas will increase their yield. Describe a controlled experiment the farmer could use to test his theory. **The farmer can play barry manilow songs to a section of his farm and isolate another section where he doesn't play the songs and observe the difference**

37. Make up three data sets with 5 numbers each that have: a. the same mean but different standard deviations.

0,2,4,6,8                      2,3,4,5,6    0,2,5,6,7

c. the same mean but different medians.

0,2,4,6,8                      1,2,3,5,9    0,2,5,6,7

d. the same median but different means.

0,2,4,6,8                      2,3,4,5,7        0,1,4,6,7

Suppose that human pregnancies have an average duration of 270 days and standard deviation of 14 days. Find the probability that a pregnancy lasts for (a) Less than 270 days  $P(X < 270) = \text{Normal cdf}(-1E99, 270, 270, 14) = 0.5$

(b) Less than 263 days  $P(X < 263) = \text{Normal cdf}(-1E99, 263, 270, 14) = 0.3085$

(c) More than 277 days  $P(X < 277) = \text{Normal cdf}(270, 1000, 270, 14) = 0.3085$

Suppose that in a youth basketball league, the average points for a team was 63 per game, with standard deviation 5. What is the probability that a team ends a game with

(a) More than 68 points  $P(X > 68) = \text{Normal cdf}(-1E99, 68, 63, 5) = 0.841$

(b) Less than 53 points  $P(X < 53) = \text{Normal cdf}(-1E99, 53, 63, 5) = 0.023$

| Math 100 (c) More than 73 points  $P(X > 73) = \text{Normal cdf}(-1E99, 73, 63, 5) = 0.977$

39. Suppose that the batting averages in baseball are normally distributed so that the mean is .258 and the standard deviation is 0.04. What is the probability that a player has a batting average of (a) More than .258?  $P(X > .258) = \text{Normal cdf}(-1E99, .258, .258, 0.04) = 0.500$

(b) More than .298?  $P(X > .298) = \text{Normal cdf}(-1E99, .298, .258, 0.04) = 0.841$

(c) More than .318?  $P(X > .318) = \text{Normal cdf}(-1E99, .318, .258, 0.04) = 0.933$